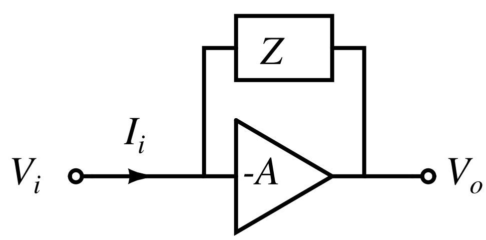
Origin of the Miller Effect

scanned by Kent H. Lundberg

Thus the apparent input capacity can become a number of times greater than the actual capacities between the tube electrodes ... — page 374

The effective input impedance of an amplifier depends on the impedance connected from input to output of the amplifier. The apparent scaling of this impedance often dominates the input impedance and frequency response of the amplifier. This effect, now commonly known as the Miller Effect, was first reported by John Miller in the following paper.

Consider the following amplifier with voltage gain $-A$, with an impedance Z connected from input to output.



Calculating the input current

$$I_i = \frac{V_i - V_o}{Z} = V_i \left(\frac{1+A}{Z} \right)$$

The Thevenin input impedance is

$$Z_{in} = \frac{V_i}{I_i} = \frac{Z}{1+A}$$

Thus a resistor or inductor connected from input to output will look a factor of $(1+A)$ times smaller (as seen from the input terminal), and a capacitor will look $(1+A)$ times larger.

References

[1] John M. Miller. Dependence of the input impedance of a three-electrode vacuum tube upon the load in the plate circuit. *Scientific Papers of the Bureau of Standards*, 15(351):367–385, 1920.

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DEPENDENCE OF THE INPUT IMPEDANCE OF A THREE-ELECTRODE VACUUM TUBE UPON THE LOAD IN THE PLATE CIRCUIT

By John M. Miller

CONTENTS

I. INTRODUCTION

In a previous paper [1] was treated the theory of the use of a three-electrode vacuum tube as an amplifier, showing the importance of the amplification constant as determining the voltage amplification of the tube and the internal resistance of the tube in the plate or output circuit as determining the alternating current flowing in that circuit. A dynamic method was given for determining these important quantities directly.

The present paper is an extension of the theory and is concerned with the characteristics of the grid or input circuit. The input impedance of the tube is of importance in determining the input power and the voltage supplied to the input terminals of the tube by the apparatus in the input circuit.

[1] Miller, Proc. I. R. E., 6, 141; 1918.

If the grid of the tube is positive with respect to the filament, there will be a flow of electrons between the filament and grid. If distortion is neglected and the frequency is so low that capacity effects are negligible, the internal input circuit is under these conditions characterized by a pure resistance and an emf in series. The value of the resistance is determined by the reciprocal of the slope of the grid-current—grid-voltage characteristic corresponding to the operating voltages. This is analagous to the internal or output resistance of the plate circuit. The internal emf which acts in the grid circuit is determined by the product of the ratio of the slopes of the grid-current—plate-voltage and grid-current—grid-voltage characteristics with the alternating voltage of the plate relative to the filament which occurs as the result of a load in the plate circuit. This again is analagous to the way in which the amplification constant and impressed alternating voltage on the grid determine the voltage acting in the plate circuit of the tube. All of these facts are implicitly contained in the equations (5) derived by M. Latour [2] in his paper on the "Theoretical Discussion of the Audion."

If the grid of the tube is negative with respect to the filament so that no appreciable electron flow takes place between these electrodes, it would appear offhand that the input impedance of the tube would be rather unimportant in determining the voltage received from the apparatus in the external circuit. In very many cases in practice, however, this is not true, and as a consequence of the capacities between the tube electrodes and connecting wires, the internal characteristics of the plate circuit of the tube and the external load in the plate circuit, the character of the input impedance of the tube markedly affects its behavior as an amplifier. The following treatment will be concerned solely with the character of the input impedance of the tube when the grid is negative with respect to the filament.

It will be shown that when the load in the plate circuit is a resistance or capacity the input impedance can be represented as a positive resistance and capacity in series. Thus the tube is not a pure voltage device, but absorbs power.

When the load is inductive the input impedance can, in many cases, be represented as a negative resistance and capacity in series. This represents regeneration through the tube itself, and is of importance in the regenerative effects and oscillations in amplifiers.

[2] M. Latour, "Electrician," 78, p. 280; 1916.

II. GENERAL THEORY OF THE DEPENDENCE OF THE INPUT IMPEDANCE UPON THE LOAD IN THE PLATE CIRCUIT

A three-electrode vacuum tube and associated circuits may be represented diagrammatically as in Fig. 1, where the continuous lines represent the circuits outside of the tube, while the dotted lines show the internal electrical characteristics of the tube. The points F, G, and P represent the three electrodes, filament, grid, and plate. The filament, grid, and plate batteries are not shown. Between filament and grid in the external circuit is applied the input emf which is an alternating voltage E_g. In the external circuit between filament and plate is inserted apparatus, such as phones, or the primary winding of a transformer, and this is designated in the figure as any impedance $Z_p = R_p + jX_p$ where

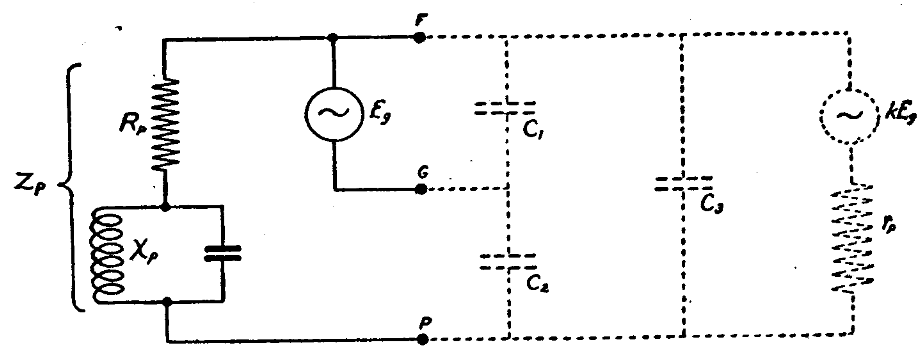

FIG. 1.—*Diagramatic representation of a vacuum tube and external circuits*

R_p is the resistance component and X_p the reactance component. The latter may be positive or negative, according as to whether it is inductive or capacitive. Within the tube the capacities between the three electrodes are represented by C_1, C_2, and C_3. In general, the capacities between the leads to these elements are not negligible and will be assumed to be lumped in correct manner with the intraelectrode capacities. Further, as shown in the earlier paper cited above, the impressed emf E_g gives rise to an internal emf $k\,E_g$ (k = amplification constant) which, acting in series with the internal output resistance r_p, is impressed between the filament and plate of the tube. In this diagrammatic representation of the tube it is assumed for simplicity that the capacities between the tube electrodes and appropriate leads are free from dielectric absorption and that the grid is maintained sufficiently negative with respect to the filament, and the insulation and vacuum are such that there is no appreciable conductive flow between the grid and filament as a result of the impressed emf E_g. Otherwise it would be necessary to assume resistances in series or in parallel with the capacities C_1, C_2, and C_3 to represent dielectric losses and an emf and series resistance in parallel with C_1, as discussed above.

The problem, then, of finding the input impedance of the tube Z_g is that of determining the current I_g which flows in the external input circuit as a result of the voltage E_g. In Fig. 2 the circuit is redrawn and the currents represented as I_g, I_1, I_2, I_3, etc.

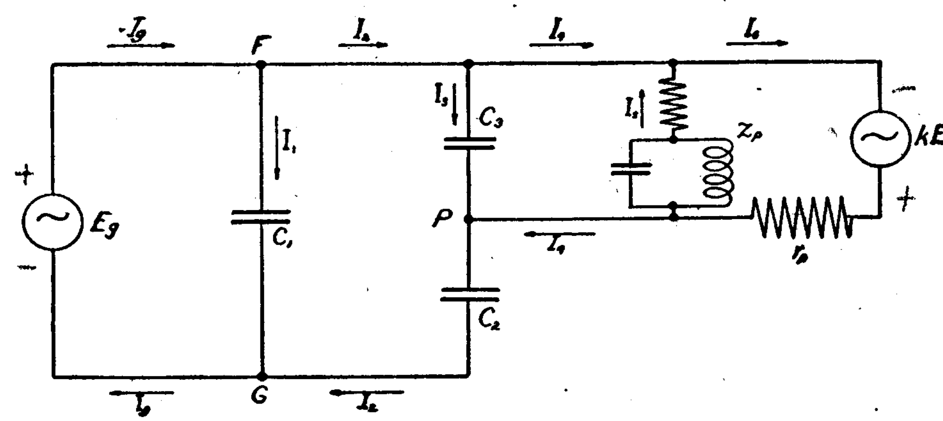

FIG. 2.—*Vacuum tube and external circuits as an electrical network*

By Kirchhoff's laws we have the following seven equations connecting the seven unknown currents, which permit us to determine I_g in terms of the quantities E_g, k, Z_p, r_p, C_1, C_2, and C_3. Thus,

$$k\,E_g = I_6\,r_p + I_5\,Z_p \tag{1}$$

$$I_6 = I_4 + I_5 \tag{2}$$

$$O = I_5\,Z_p + \frac{I_3}{j\,\omega\,C_3} \tag{3}$$

$$I_2 = I_3 + I_4 \tag{4}$$

$$O = \frac{I_3}{j\,\omega\,C_3} + \frac{I_2}{j\,\omega\,C_2} - \frac{I_1}{j\,\omega\,C_1} \tag{5}$$

$$I_g = I_1 + I_2 \tag{6}$$

$$E_g = \frac{I_1}{j\,\omega\,C_1} \tag{7}$$

Eliminating I_1, I_2, I_3, I_4, I_5, and I_6 between these equations and writing $z_g=\frac{E_g}{I_g}$ we obtain the following expression for the input impedance:

$$z_g=\frac{r_p\,(C_2+C_3)-\frac{j}{\omega}-\frac{j}{\omega}\frac{r_p}{Z_p}}{k\,C_2+C_1+C_2+\frac{r_p}{Z_p}(C_1+C_2)+j\,\omega\,r_p\,(C_1\,C_2+C_1\,C_3+C_2\,C_3)} \tag{8}$$

Substituting $Z_p=R_p+j\,X_p$ (9)

in (8) we obtain the equation

$$z_g=\frac{a+j\,b}{c+j\,d} \tag{10}$$

where

$$a=R_p\,(C_2+C_3)+\frac{X_p}{\omega r_p}$$

$$b=X_p\,(C_2+C_3)-\frac{R_p}{\omega\,r_p}-\frac{1}{\omega} \tag{11}$$

$$c=\frac{R_p}{r_p}\,(k\,C_2+C_1+C_2)+C_1+C_2-\omega\,X_p\,(C_1\,C_2+C_1\,C_3+C_2\,C_3)$$

$$d=\frac{X_p}{r_p}\,(k\,C_2+C_1+C_2)+\omega\,R_p\,(C_1\,C_2+C_1\,C_3+C_2\,C_3)$$

And if z_g is separated into resistance and reactance components r_g and x_g, we have

$$z_g=r_g+j\,x_g \tag{12}$$

$$r_g=\frac{a\,c+b\,d}{c^2+d^2} \tag{13}$$

$$x_g=\frac{b\,c-a\,d}{(c^2+d^2)} \tag{14}$$

If X_p is negative, corresponding to a capacity reactance in the plate circuit, we have the following terms in the numerator of r_g:

Positive terms	Negative terms
$\omega X_p R_p\,(C_2+C_3)\,(C_1C_2+C_1C_3+C_2C_3)$	$\omega X_p R_p\,(C_2+C_3)\,(C_1C_2+C_1C_3+C_2C_3)$
$\frac{X^2_p}{r_p}\,(C_2+C_3)\,(kC_2+C_1+C_2)$	$\frac{X^2_p}{r_p}\,(C_1C_2+C_1C_3+C_2C_3)$
$\frac{R_pX_p}{\omega r_p^2}\,(kC_2+C_1+C_2)$	$\frac{R_pX_p}{\omega r_p^2}\,(kC_2+C_1+C_2)$

Positive terms	Negative terms
$\frac{X_p}{\omega r_p}(kC_2+C_1+C_2)$	$\frac{X_p}{\omega r_p}(C_1+C_2)$
$\frac{R^2_p}{r_p}(C_2+C_3)(kC_2+C_1+C_2)$	$\frac{R^2_p}{r_p}(C_1C_2+C_1C_3+C_2C_3)$
$R_p(C_2+C_3)(C_1+C_2)$	$R_p(C_1C_2+C_1C_3+C_2C_3)$

It is evident from inspection that the positive terms will exceed numerically the negative terms when $X_p \lesseqgtr 0$, and since the denominator of r_g is always positive, r_g must always be positive.

The resistance component of the input impedance of a three-electrode vacuum tube is always positive, and hence the input absorbs power if the load in the plate circuit is capacitive or a pure resistance even when the grid is negative with respect to the filament.

In the succeeding treatment it will be shown that the input impedance of the tube may be equivalent to a considerable capacity with a high resistance in series, in which case the absorption of power in the input of the tube becomes very large.

If, however, the load is inductive ($X_p > 0$), the terms above which contain X_p will change sign. The numerator of r_g will then become

$$R_pC_2^2+\frac{R_p^2}{r_p}(kC_2^2+C_2^2+kC_2C_3)+\frac{X_p^2}{r_p}(kC_2^2+C_2^2+kC_2C_3)-\frac{X_p}{\omega r_p}(kC_2).$$

Hence r_g will be negative if

$$\frac{kX_p}{\omega r_p}>\frac{X_p^2}{r_p}(kC_2+C_2+kC_3)+\frac{R_p^2}{r_p}(kC_2+C_2+kC_3)+R_pC_2. \qquad (15)$$

The resistance component of the input impedance of a three-electrode vacuum tube can be negative and the tube will supply power to the external input circuit; i. e., regenerate, if the load in the plate circuit is inductive.

This explains the regenerative effect of an inductive load previously noted by Armstrong,[3] and also the regenerative effects and oscillations in amplifiers, which can occur even when there is no electrostatic or electromagnetic coupling between the input and output circuits other than through the tube itself.

The dependence of the regenerative action upon the inductive load in the plate circuit will be treated theoretically and experimentally in a succeeding section.

[3] Armstrong, E. H., Proc. I. R. E., 3, p. 215, 1915; in particular Fig. 10, on p. 220.

III. INPUT IMPEDANCE FOR THE CASE OF A PURE RESISTANCE LOAD IN THE PLATE CIRCUIT

We will first consider the case where the load in the plate circuit is a pure resistance; that is, $Z_p = R_p$. Equation (8) then becomes

$$z_g = \frac{r_p(C_2+C_3) - \frac{j}{\omega}\left(1+\frac{r_p}{R_p}\right)}{kC_2 + \left(1+\frac{r_p}{R_p}\right)(C_1+C_2) + j\,\omega\, r_p\,(C_1\,C_2 + C_1\,C_3 + C_2\,C_3)} \qquad (16)$$

If we let

$$\begin{aligned} a &= r_p\,(C_2+C_3) \\ b &= \left(1+\frac{r_p}{R_p}\right) \\ c &= kC_2 + \left(1+\frac{r_p}{R_p}\right)(C_1+C_2) \\ d &= r_p\,(C_1\,C_2 + C_1\,C_3 + C_2\,C_3) \end{aligned} \qquad (17)$$

$$\text{Then } z_g = \frac{\left(a - \frac{jb}{\omega}\right)(c + j\omega d)}{c^2+\omega^2\,d^2} = \frac{ac+bd}{c^2+\omega^2 d^2} + \frac{1}{j\omega\frac{(c^2+\omega^2 d^2)}{(bc+a\omega^2 d)}}$$

If the input impedance is represented by an apparent resistance r_g in series with an apparent capacity c_g, the values of these quantities are given by

$$\begin{aligned} r_g &= \frac{a\,c+b\,d}{c^2+\omega^2 d^2} \\ c_g &= \frac{c^2+\omega^2 d^2}{b\,c+a\omega^2\,d} \end{aligned} \qquad (18)$$

The relative importance of the quantities involved may be expressed by $b > > a$, $c > > d$. Hence at low frequencies (in general for $\omega < 10^6$)

$$\begin{aligned} r_g &= \frac{a\,c+b\,d}{c^2} \\ c_g &= \frac{c}{b} \end{aligned} \qquad (19)$$

For $R_p = 0$; $r_g = 0$; $c_g = C_1 + C_2$. Under these conditions the plate circuit constitutes a short circuit between filament and plate, eliminating the capacity C_3 and putting C_1 and C_2 in parallel between grid and filament.

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As R_p increases relative to r_p, both r_g and c_g increase. r_g can increase to nearly the order of r_p. The variation in c_g can be expressed by the equation

$$c_g = C_1 + C_2 + C_2 \left(\frac{k\ R_p}{r_p + R_p} \right) \tag{20}$$

From this it appears that the capacity C_2 between grid and filament is important in increasing the apparent input capacity. The maximum increase (for $R_p > > r_p$) is $k\ C_2$. It is of interest to note that the quantity $\frac{k\ R_p}{r_p + R_p}$ is, under the assumed frequency conditions, the ratio of the voltage across R_p to the input

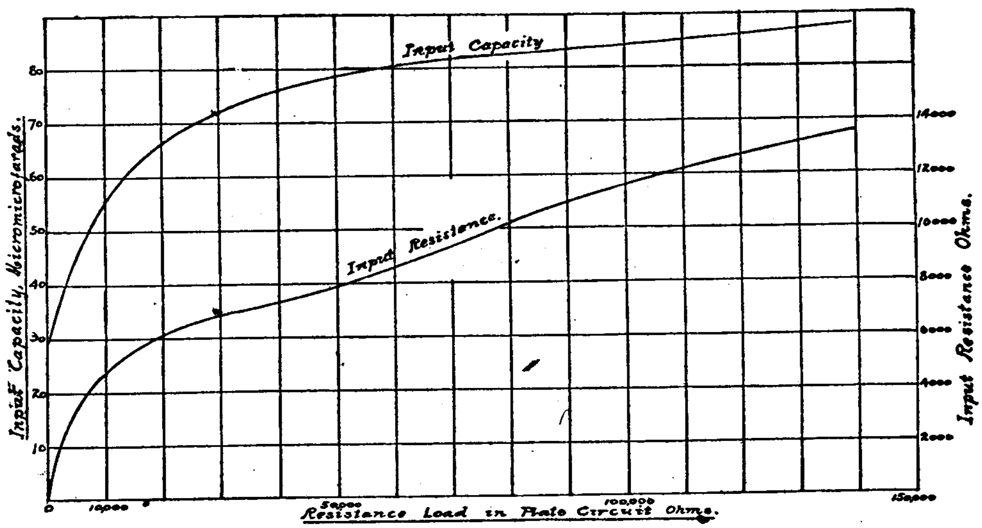

FIG. 3.—*Variation of input characteristics with resistance load in the plate circuit*

voltage E_g and hence determines the voltage amplification per stage of a resistance coupled amplifier. Thus the apparent input capacity can become a number of times greater than the actual capacities between the tube electrodes, and since the apparent input resistance can also become very high, the dissipation of power in the input circuit of the tube may be considerable, even when the grid is negative with respect to the filament.

When the frequency is so high that the terms containing ω^2 become important, these resistance and capacity effects become less marked. For very high frequencies

$$r_g \doteq 0$$

$$c_g = \frac{C_1\ C_2 + C_1\ C_3 + C_2\ C_3}{C_2 + C_3}$$

This latter is the capacity of C_2 and C_3 in series and paralleled by C_1—i. e., the capacity between filament and grid with the plate circuit open. At these frequencies, however, the voltage across the resistance R_p is reduced, because of capacity effects and approaches zero with increasing frequency. Fig. 3 shows the variation of r_g and c_g with the load R_p for a particular tube of the J or VT–1 type for wave lengths longer than about 2000 m.

IV. EXPERIMENTAL DETERMINATIONS WITH A PURE RESISTANCE LOAD IN THE PLATE CIRCUIT

1. DETERMINATION OF k, r_p AND $\frac{k R_p}{R_p+r_p}$

A dynamic method for determining k and r_p, which was described in an earlier paper,[4] was utilized in these measurements. Since with a constant plate battery the actual voltage on the tube is reduced as a result of the drop in voltage across R_p, the determinations were so made that values of k, r_p, and $\frac{k R_p}{R_p+r_p}$ could be obtained which corresponded to the actual voltage on the plate for a given R_p. This was effected by obtaining curves for k and r_p for varying plate voltages, and then, with a constant plate battery, the actual voltage on the tube was determined for different values of R_p by making readings of the plate current and computing the voltage drop.

Two of the tubes used in the experiments and the electrical data of their use are described in Table 1.

TABLE 1

Type.	Plate voltage	Filament current	Grid voltage
J or VT–1	40	1.1	−1.5
VT–3	40	0.2	−1.5

Figs. 4 and 5 give the curves showing the dependence of k, r_p, and $\frac{k R_p}{R_p+r_p}$ upon the load R_p for these two tubes.

2. DETERMINATION OF C_1, C_2, AND C_3

A series-resistance capacity bridge was used to measure the tube capacities, using an amplifier and phones as a balance indicator. A ground connection was put on a third arm of the bridge, and this was adjusted so as to bring the detecting arm of the bridge

[4] See reference 1.

at ground potential. The measurements were made at about 1000 cycles with a few tenths of a volt impressed on the bridge. Under these conditions the bridge was sensitive to one-tenth

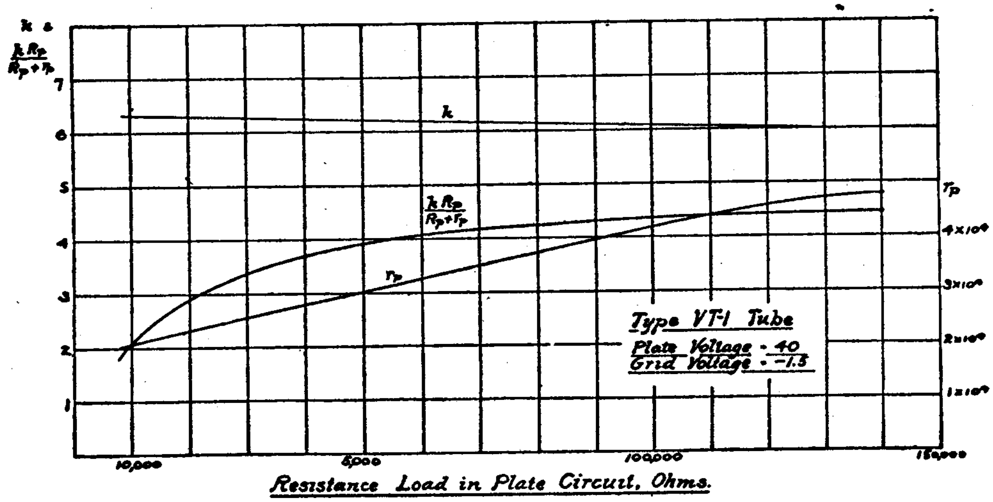

FIG. 4.—*Amplification constant, voltage amplification and internal plate circuit resistance with varying resistance load. VT–1 tube*

micromicrofarad. One arm of the bridge contained a variable air condenser of 250 micromicrofarads capacity, across which the capacities to be measured were connected and determined by the necessary change in the variable to maintain the bridge balance.

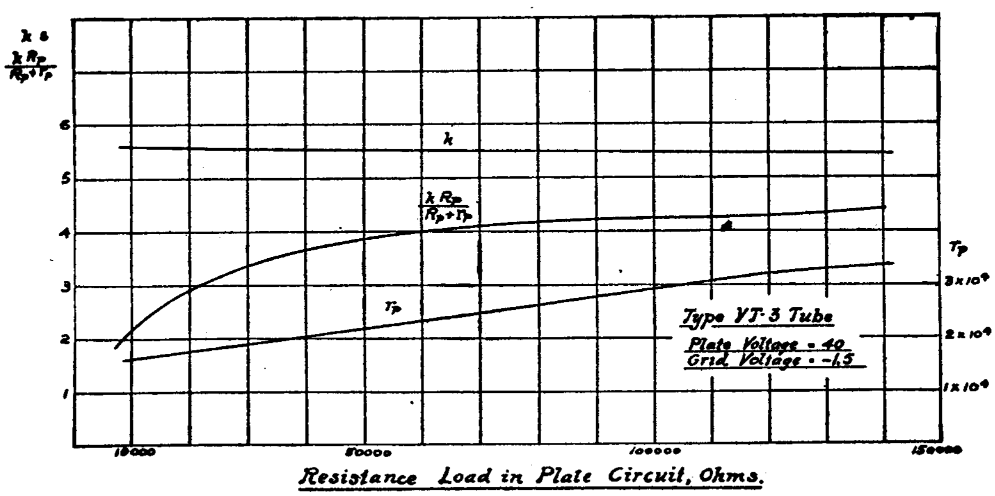

FIG. 5.—*Amplification constant, voltage amplification and internal plate circuit resistance with varying resistance load. VT–3 tube*

The filament, plate, and grid batteries were connected directly to the negative filament terminal, and this point was likewise connected to the ground potential part of the bridge. Those portions of the connecting leads to the tube electrodes which followed the potentials of the electrodes themselves were considered as part of the electrodes and included in the capacity measurements. The tube socket was a Signal Corps receiving-tube socket and its capacities were also included.

The capacities C_1, C_2, and C_3 were separately determined in the following manner:

(*a*) Connect G and P together and measure capacity to F. This short-circuits C_2 and gives C_1+C_3.

(*b*) Connect G and F and measure to P. This gives C_2+C_3.

(*c*) Connect F and P (i. e. $R_p=O$) and measure to G. This gives C_1+C_2.

From these observations, then

$2C_1 = (a) + (c) - (b)$

$2C_2 = (b) + (c) - (a)$

$2C_3 = (a) + (b) - (c)$

The values of the capacities in micromicrofarads as measured for the two tubes mentioned previously were found to be as shown in Table 2.

TABLE 2

Type	C_1+C_3	C_2+C_3	C_1+C_2	C_1	C_2	C_3
VT-1	28.2	26.6	27.9	14.7_5	13.1_5	13.4_5
VT-3	24.1	18.8	18.9	12.1	6.8	12.0

These capacity values are considerably increased because of the tube socket and leads used in the experiments.

3. DETERMINATION OF c_g

The apparent input capacity c_g for different resistance loads was determined in the same way as C_1+C_2 in (2) above, excepting that the resistance R_p was inserted in the plate circuit of the tube.

4. COMPARISON OF OBSERVED AND COMPUTED RESULTS

In Tables 3 and 4 the various resistance loads which were inserted in the plate circuit are given in the first column and in the other columns the calculated and experimentally observed values of the input capacity c_g are given in micromicrofarads.

TABLE 3.—VT-1 Tube

R_p, ohms	Input capacity	
	Computed	Observed
0	....	27.9
8000	51.4	49.0
16 000	64.5	61.5
49 400	78.9	76.1
97 000	84.2	84.3
139 000	86.1	87.6

TABLE 4.—VT-3 Tube

R_p, ohms	Input capacity	
	Computed	Observed
0	....	18.9
8200	31.8	32.4
18 500	38.1	40.1
49 800	45.1	46.9
98 500	47.5	51.2
140 500	49.0	53.1

To show the importance of the capacity C_2 in determining c_g a separate series of measurements were carried out in which the capacity C_2 was increased by connecting a small condenser between the grid and plate. A different tube of the VT–1 type was used in these measurements and the resistance R_p was 30 000 ohms throughout, leading to a value of $\frac{k R_p}{R_p + r_p}$ of 3.29. Measurements of the apparent input capacity were made with C_2 increased by zero, 17.5, and 34.3 micromicrofarads. The values of C_2 were, then, 11.8, 29.3, and 46.1 micromicrofarads, the value of C_1 was 12.2 micromicrofarads, the values of $C_1 + C_2$ were 24.0, 41.5, and 58.3 micromicrofarads. The values of c_g as calculated from the formula $c_g = C_1 + C_2 + C_2 \frac{k R_p}{r_p + R_p}$ for the three cases were 62.8, 137.9, and 210.0 micromicrofarads. The experimentally observed values were 64.3, 138.6, and 205.4 micromicrofarads, showing an agreement of about 2 per cent.

It was found to be impossible to check the values of r_g experimentally at the frequencies used in the bridge measurements because of dielectric absorption in the tube capacities. At these low frequencies the dielectric losses introduce effective resistances which are many times greater than those given by the expression for r_g, which does not take dielectric losses into account. The measurements can no doubt be made at radio frequencies, but are rendered somewhat difficult because of the limited input voltage which can be applied to the tube if the grid is to remain at all times negative with respect to the filament. The dielectric losses in the tube capacities are doubtless important in the use of tubes at long waves, and should be taken into account in the design of tube bases and sockets.

V. INPUT IMPEDANCE FOR THE CASE OF AN INDUCTIVE LOAD IN THE PLATE CIRCUIT

In case the load in the plate circuit is an inductance L_p and resistance R_p and the input impedance of the tube is represented by a series resistance r_g and capacity c_g, we obtain from equations (11), (13), and (14) the following:

$$r_g = \frac{a\,c+b\,d}{c^2+d^2} \qquad (21)$$

$$c_g = \frac{(c^2+d^2)}{\omega(ad-bc)} \qquad (22)$$

where

$$\begin{aligned}
a &= R_p(C_2+C_3) + \frac{L_p}{r_p} \\
b &= \omega L_p(C_2+C_3) - \frac{R_p}{\omega r_p} - \frac{1}{\omega} \\
c &= \frac{R_p}{r_p}(kC_2+C_1+C_2) + C_1 + C_2 - \omega^2 L_p(C_1C_2+C_1C_3+C_2C_3) \\
d &= \frac{\omega L_p}{r_p}(kC_2+C_1+C_2) + \omega R_p(C_1C_2+C_1C_3+C_2C_3)
\end{aligned} \qquad (23)$$

As already pointed out above in expression (15), the numerator of r_g, and hence r_g itself, will be zero or negative when

$$\frac{kL_p}{r_p} \geqq \frac{\omega^2 L_p^2}{r_p}(kC_2+C_2+kC_3) + \frac{R_p^2}{r_p}(kC_2+C_2+kC_3) + R_pC_2 \qquad (24)$$

The equality sign determines the values of L_p, for which the input resistance is zero. If R_p is large, the solutions for L_p at a given frequency may be imaginary, in which case no inductive load can make the input resistance negative.

Curves showing the variation in the input resistance and input capacity with the inductance in the plate circuit are given in

Figs. 6 and 7 for various values of R_p. These were computed, using formulas (21), (22), and (23) for a frequency given by $\omega = 2 \times 10^6$, and assuming the constants $k = 6$, $C_1 = C_2 = C_3 = 10^{-11}$ and $r_p = 2 \times 10^4$, which are approximately those of a VT–1 tube.

If we assume that the resistance in the plate circuit is so low compared to the reactance of L_p that the terms containing R_p are negligible, the inequality of (24) reduces to

$$\omega^2 L_p \left(C_2 + C_3 + \frac{C_2}{k} \right) < 1 \qquad (25)$$

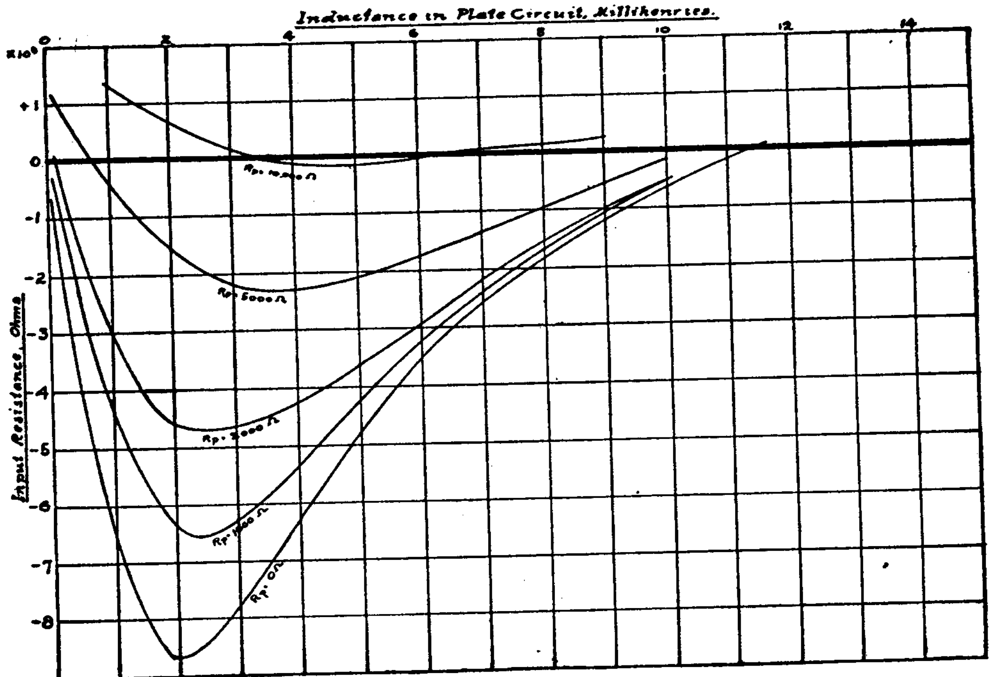

FIG. 6.—*Negative input resistance caused by inductive load in plate circuit*

This shows that the combination of inductive load plus the tube capacities must still be an inductive reactance in order to have regeneration, and determines the highest frequency with a given inductance L_p or the highest value of L_p at a given frequency at which regeneration can occur. At low values of L_p, or at low frequencies where $\omega^2 L_p (C_1, C_2, \text{or } C_3)$ is small compared to unity, and assuming R_p is small compared to r_p or ωL_p, the only term in the numerator of r_g which is of importance is $-\frac{L_p}{r_p}(kC_2)$. From (21) and (23) it is seen that the denominator of r_g reduced to $(C_1 + C_2)^2$. Hence the value of the input resistance is given by

$$r_g = -\frac{L_p}{r_p} \frac{(k\, C_2)}{(C_1 + C_2)^2} \qquad (26)$$

From (22) and (23) it can be seen that under the above assumptions the input capacity is given by

$$c_g = C_1 + C_2 \qquad (27)$$

Under these conditions, therefore, the input impedance of a tube consists of a negative resistance proportional to the inductance in the plate circuit in series with a constant capacity. This corresponds to the portion of the curves of Figs. 6 and 7 for $R_p = 0$ and low L_p.

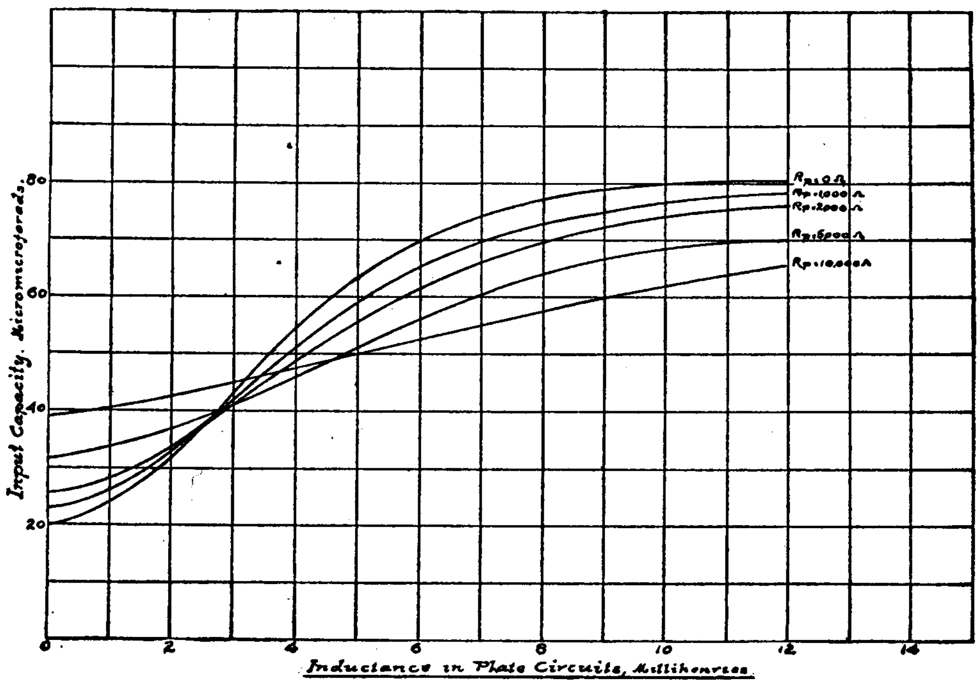

FIG. 7.—*Variation in input capacity with inductive load*

The magnitude of the regenerative effect produced by the negative input resistance will depend upon the constants of the external input circuit. The effect will be to reduce or neutralize the positive resistance of the external circuit. In general an oscillatory circuit is connected to the input of the tube and the apparent resistance of this circuit is reduced as a result of the regenerative action. When r_g and c_g are such as completely to neutralize the resistance of that circuit, oscillations will take place.

In the case of an amplifier this input circuit may be a transformer. The complete input circuit will be as shown in Fig. 8, where L and C represent the coil and condenser of the oscillatory circuit, of which the resistance is R. The input characteristics of the tube are represented by r_g and c_g. The reduction in the resistance of the oscillatory circuit which results when r_g is negative, can be calculated as follows. Since the reactance of c_g is ordinarily very large compared to the numerical value of r_g, the current I in the oscillatory circuit will divide between C and the parallel branch containing c_g in proportion to the capacities C and c_g. Hence the current

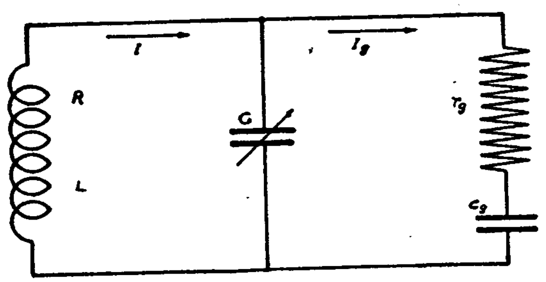

FIG. 8.—*External grid circuit and input impedance of tube*

I_g flowing into the grid of the tube and through the resistance r_g is given by

$$I_g = I \frac{c_g}{C+c_g} \tag{28}$$

The power dissipated in r_g will be $P_g = I_g^2 r_g = I^2 \left(\frac{c_g}{C+c_g}\right)^2 r_g$ (29)

This will be negative when r_g is negative, thus representing a generation of power. The power dissipated in the resistance R is

$$P_R = I^2 R \tag{30}$$

and the total power

$$P = P_R + P_g = I^2 \left[R + \left(\frac{c_g}{C+c_g}\right)^2 r_g\right] \tag{31}$$

Thus when r_g is negative the reduction in the circuit resistance will be given by

$$\Delta R = \left(\frac{c_g}{C+c_g}\right)^2 r_g \tag{32}$$

In Fig. 9 are plotted curves of received signal against the inductance in the plate circuit, assuming the same tube constants and frequency as in Figs. 6 and 7, that the resistance in the plate circuit is negligible ($R_p = 0$) and that the capacity C of Fig. 8 is 0.0015 microfarad. Three curves are shown corresponding to circuits of 7, 9, and 10 ohms resistance. The received signal is taken to be proportional to the reciprocal of the circuit resistance as reduced by the regenerative effect. The curves for the 7 ohm circuit run to infinity, indicating complete neutralization of the circuit resistance and hence oscillations. The curves for the 9 and 10 ohm circuits are quite similar to that given by Armstrong.[5]

For low values of L_p we find by substituting the values of r_g and c_g from (26) and (27) in (32)

$$\Delta R = \frac{k\, L_p\, C_2}{r_p\, (C + C_1 + C_2)^2} \tag{33}$$

The regenerative effects are increased by increasing L_p, decreasing C, or by connecting a condenser between grid and plate so as to increase C_2 when C_2 is small compared to C.

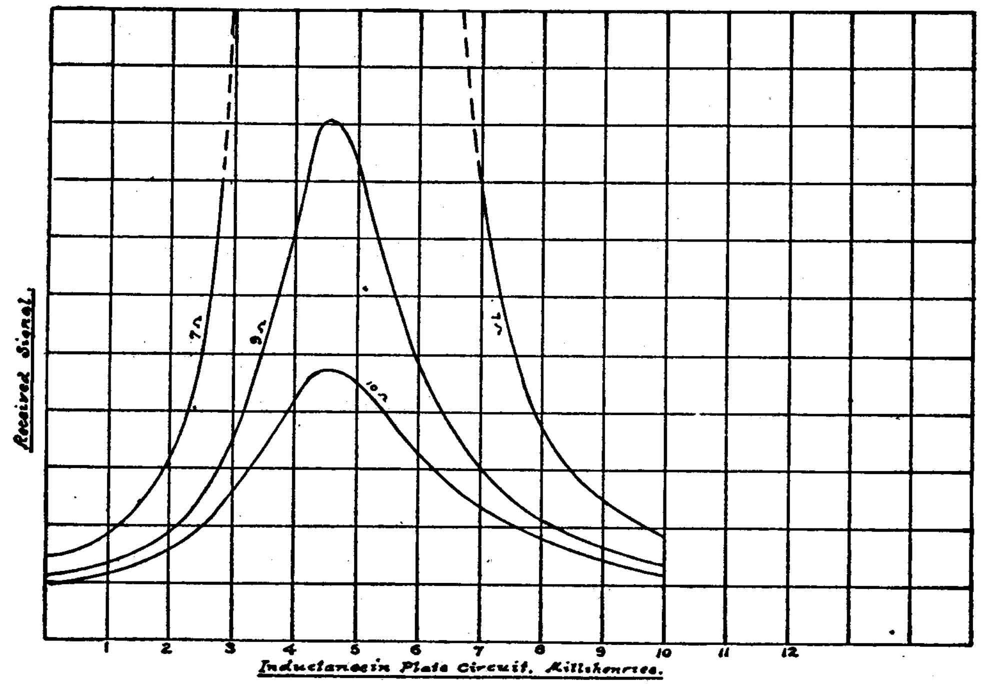

FIG. 9.—*Variation of received signal with inductive load in the plate circuit*

VI. EXPERIMENTAL DETERMINATIONS WITH AN INDUCTIVE LOAD

Expression (33) was checked by measuring at radio frequencies the reduction in resistance of an oscillatory circuit connected to the input terminals of the tube when different inductances of known value were inserted in the plate circuit.

1. DETERMINATION OF THE TUBE CONSTANTS

A type J or VT–1 tube was used with 40 volts on the plate, −1.5 on the grid, and a filament current of 1.1 amperes. The

[5] Armstrong, loc. cit.

tube constants were measured, as outlined in Section IV above, and were found to be as follows for the tube used: $k=7.2$, $r_p=29\,300$, $C_2=12.2$, and $C_1+C_2=25.7$ micromicrofarads.

2. MEASUREMENT OF THE OSCILLATORY CIRCUIT RESISTANCE

The oscillatory circuit was coupled to a driving circuit, and its resistance was measured at 900 m. wave length by the resistance variation method. The capacity C was 1675 micromicrofarads. The current indications were obtained with a vacuum thermocouple of 4.8 ohms resistance and a sensitive wall galvanometer. With this value of the capacity and frequency sufficient measuring current was obtained without impressing more than one volt across the condenser or on the input of the tube. Measurements made with the tube disconnected from the condenser C, and then connected, but with no inserted inductance in the plate circuit, showed that the dielectic losses in the tube capacities were not appreciable at this frequency. The resistance of the oscillatory circuit was then determined with 254 and 680 microhenries inductance in the plate circuit.

3. COMPARISON OF OBSERVED AND COMPUTED RESULTS

The theoretical reduction in the circuit resistance was computed from formula (33); the value of the factor $\frac{k C_2}{r_p(C+C_1+C_2)^2}$ as calculated from the tube constants and capacity C being 1.037×10^3. The results are compared in Table 5. In the first column are given the values of the inductive load in microhenries, in the second column the corresponding observed circuit resistances, in the third column the observed reduction in the circuit resistance, and in the fourth column the reduction in circuit resistance as computed by formula (33).

TABLE 5

Inductance, microhenries	Circuit resistamce, ohms	Reduction in circuit resistance	
		Observed	Computed
0	6.58	....	....
254	6.32	0.26	0.26
680	5.88	.70	.70

VII. INPUT IMPEDANCE FOR THE CASE OF A CAPACITY LOAD IN THE PLATE CIRCUIT

The equations of the input impedance for a capacity load can likewise be derived readily from equations (11), (13), and (14). In this case the input resistance will always be positive, so that the input absorbs power. Thus the presence of phones in the plate circuit of a tube may cause a dissipation of power in the input, because of the phones having a capacity reactance at high frequencies.

VIII. SUMMARY OF RESULTS

1. Because of the capacities between the elements of a three-electrode vacuum tube, the input impedance of the tube depends upon the nature of the load in the plate circuit of the tube.

2. Even when the grid of the tube is negative with respect to the filament, the input impedance can be such as to absorb considerable power from the input circuit, This occurs when the load in the plate circuit is a resistance or capacity reactance.

3. When the load in the plate circuit is inductive, the input impedance can be characterized by a negative resistance, in which case regeneration or oscillations can occur as a result of coupling through the tube itself.

In conclusion the author desires to express his indebtedness to the Signal Corps, who requested and supported this investigation, and to Miss Dora E. Wells, of the Bureau of Standards, who performed most of the experimental work. The above results were communicated to the Signal Corps in reports dated April 8 and April 30, 1919, and are published with their approval.

WASHINGTON, June 11, 1919.